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# A GENERALIZED NON-REGRESSION TECHNIQUE FOR EVALUATING THE FRACTAL DIMENSION OF RASTER GIS LAYERS CONSISTING OF NON-SQUARE CELLS

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**Abstract.** Natural landscapes often reveal extremely complex patterns that can only be very roughly characterized by methods of Euclidean geometry. In contrast, fractals can be applied to a variety of landscape ecology problems because they conveniently describe many of the irregular, fragmented patterns found in nature. This paper focuses on a fractal-based measure of landscape complexity for grid-based GIS layers. A non-regression technique for measuring the distribution of diversity within a raster database consisting of square cells is generalized to incorporate any regular shaped grid cell (e.g. regular polygon, rectangle) that forms a continuous, fully tessellated grid.

## 1. Introduction

The effect of landscape structure on ecological processes is a key problem area in landscape ecology. Terrestrial landscapes consist of mosaics of individual parcels or patches of different land cover types. Landscape structure is largely determined by the origin, number and size of patches. However, the spatial juxtaposition of patches may be equally important (Forman & Godron 1986). Bailey et al. (1978) observe that "the relationship between components of landscape and physical and biological process is almost always through spatial pattern or structure rather than through composition alone." Therefore, methods of analyzing and interpreting landscape structure are becoming increasingly important for ecological studies (Turner 1990). A natural way to describe and compare landscape structure is by measuring the shape of patches within a landscape (De Cola 1989). Indices of shape have a long history (Haggett et al. 1977), but until the early 80's patch shape was not incorporated as a variable in landscape ecology models or theory because a geometrical framework was lacking. Simple Euclidean methods have been shown to be inadequate for treating complex shapes (Rex & Malanson 1990).

Since Mandelbrot (1967, 1982) introduced the concept of fractals, researchers have turned to fractal geometry to incorporate patch shape in ecological models and theory (e.g., the classical work of Krummel et al. 1987, De Cola 1989, Rex & Malanson 1990). These authors used fractal geometry to estimate landscape complexity as a function of patch shape by computing the slope of a regression line between the natural logarithm of perimeter and area pairs calculated for each or all cover types of interest. Because this method employs regression analysis, it is subject to spurious results if insufficient perimeter and area pairs are available to calculate the regression. When an insufficient number of pairs are available, a single fractal value can be determined for the entire landscape mosaic using the perimeter-area relationship (Gardner et al. 1987, Olsen et al. 1993, Mc Garigal & Marks 1995).

A classical law of geography and landscape ecology holds that everything in a landscape is interrelated, but near ecosystems are more related than distant ones (Forman 1995). In particular, the components of ecological systems such as specific composition, biotic movement or fluxes of nutrient, water, and energy are spatially punctuated and more greatly affected by neighbouring ecosystems. Therefore, in most applications, researchers working at the landscape level need to understand the distribution of diversity over a given landscape (i.e., what areas within the landscape are more diverse than others), not just the overall diversity of the landscape (Olsen et al. 1993).

In order to determine the distribution of diversity within a grid-based landscape, reduced sub-areas of interest from the larger landscape need to be examined. Unfortunately, using the traditional regression method for computing a fractal index for small sub-landscapes gives rise to a major problem. As the landscape extent decreases, the number of patches in the landscape also declines, reducing the sample size for statistical estimation. To resolve this shortcoming, Olsen et al. (1993) proposed a non-regression technique for calculating the fractal dimension for small sub-landscapes as an alternative intended for use within a grid-based GIS layer consisting of nominally scaled (i.e., classified) square cells.

The square pixel has become dominant in grid-based GIS compared to any other method of tessellation because, particularly in GIS applications related to broad-scale natural resource management, classified raster data sets are generally derived from satellite thematic mapping based on square cells (Hinton 1996, Fisher 1997). However, other tessellated GIS layers composed of non-square cells (e.g. rectangles, regular polygons) are sometimes utilized. For example, the 1980/84 and 1990/94 forest inventories of Belgium have been conducted as a systematic survey over a 1000 by 500 m rectangular grid (Rondeux 1994), while the Environmental Monitoring and Assessment Program (EMAP) of Pennsylvania used a raster databases consisting of hexagonal grid cells of 635 $km^2$ (Johnson et al. 1996). In order to analyze the land cover changes that occurred in the Kissimmee/Everglades Basin (Florida) from 1900 to 1973, Costanza & Maxwell (1994) used a set of three land use maps tessellated into rectangular cells of 625 by 833 m.

The methodology presented here expands on the work of Olsen et al. (1993) and addresses the need to evaluate the distribution of landscape diversity within a grid-based GIS layer consisting of any regular shaped grid cell which yield an exhaustive tessellation of a plane, such that the plane is completely covered with cells and that no part of the plane is covered more that once (Figure 1).

## 2. Fractal dimension of a sub-landscape consisting of square cells

For completeness and as a comparison, the non-regression technique for calculating the fractal dimension of a single tessellated landscape patch, as described by Olsen et al. (1993), is included. Next, the computation of the fractal dimension for the whole sub-landscape is shown. Note that in the case of a nominally scaled raster database, a patch may be defined as a connected set of cells assigned to the same class. Also, the term sub-landscape refers to a sampled area from a larger grid-based landscape.

The perimeter $P$ of a landscape patch is related to the area $A$ of the same patch by the basic fractal relationship (Johnson et al. 1995):

$$P = k \times A^{D/2} \tag{1}$$

where $D$ is the fractal dimension = 1 (a single cell is our simplest case), $k$ is the constant of proportionality for a grid cell, $A = 1$ cell area, $P = 4$ cell lengths.

To avoid using the regression technique, the constant of proportionality $k$ for Equation 1 needs to be calculated (note that using a constant of proportionality in the regression technique serves no mathematical purpose since the logarithm of $k$ is the intercept calculated for the regression line). Rearranging Equation 2 solves for $k = 4$.

$$P = 4 \times A^{D/2} \tag{2}$$

and

$$D = 2 \ln (P/4)/\ln(A) \tag{3}$$

where $A$ is total patch area, $P$ is the total patch perimeter, and $D$ is the fractal dimension.

Equation (3) became the relationship for calculating the fractal dimension of each patch separately. Note that major problems occur for single-cell patches (i.e., those with $A$ = 1), as they yield an undefined result in Equation 3. However, the objective here is the fractal dimension of the entire sub-landscape and not single patches. To accomplish this, once each patch within the sub-landscape is identified in terms of area and perimeter (Figure 2), the sub-landscape fractal dimension is calculated on the total area and perimeter units contained by the sub-landscape using Equation 3 (Kenkel & Walker 1996).

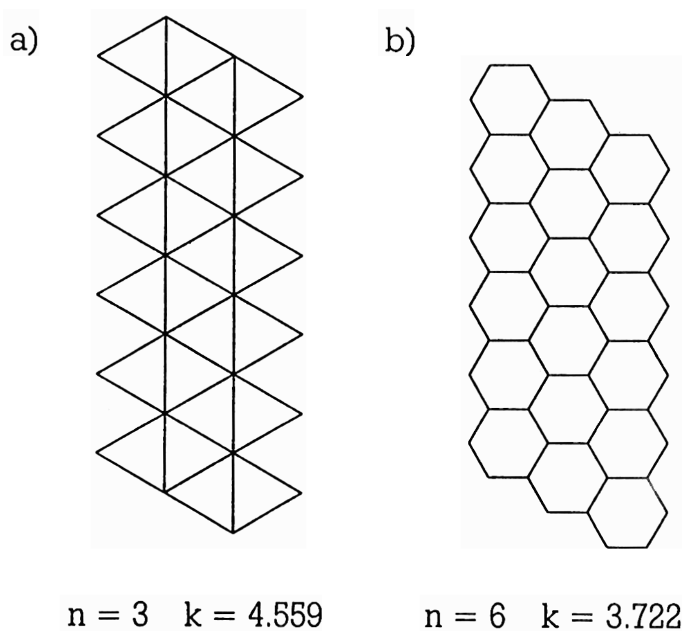
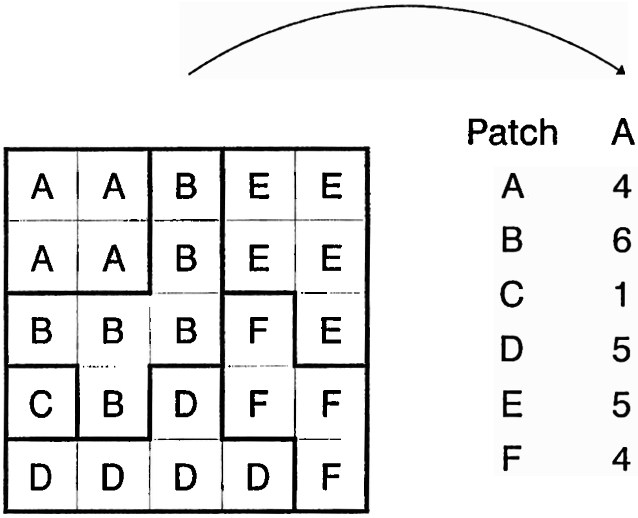



**Figure 1. Example of grids consisting of a) triangular and b) hexagonal cells. Like rectangular cells, both kinds of cell yield an exhaustive tessellation of a plane such that the plane is completely recovered with cells and no part of the plane is covered more than once.**




| Patch | A | P |
|---|---|---|
| A | 4 | 8 |
| B | 6 | 14 |
| C | 1 | 4 |
| D | 5 | 12 |
| E | 5 | 10 |
| F | 4 | 10 |
| Total | 25 | 58 |

Landscape D = 2 ln(58/4)/ln25 = 1.662

**Figure 2. Example calculation using the non regression technique of Equation 5. This 5 by 5 cell grid represents a small subset within the original landscape for which local diversity was measured.**

## 3. Fractal dimension of a sub-landscape consisting of any regular shaped cell

To modify the previous non-regression technique to handle any nominally scaled patch composed of regular shaped cells which form a continuous grid, the constant of proportionality $k$ in Equation 2 needs to be recalculated. Recall that for any regular polygon:

$$A = \frac{1}{4} n l^2 \cot\left(\frac{\pi}{n}\right) = 1 \qquad (4)$$

with $l$ = length of one side and $n$ = number of sides

We can then calculate the perimeter $P$ in terms of $l$:

$$l = \sqrt{\frac{4}{n} \times \frac{1}{\cot(\pi/4)}} \qquad (5)$$

giving

$$P = nl = \sqrt{n/\cot(\pi/n)} \qquad (6)$$

For example, for hexagonal pixels ($n = 6$), we define:

$$P = 2\sqrt{6/\cot(\pi/6)} = k \times A^{D/2}$$

and

$$k = 3.722$$

**Table 1. Proportionality constants ($k$) for regular polygons forming a regular tessellation of the plane.**

| Polygon | Number of sides | $k$ |
|---|---|---|
| Equilateral triangle | 3 | 4.559 |
| Square | 4 | 4.000 |
| Hexagon | 6 | 3.722 |

By introducing the constant of proportionality $k$ in Equation 3 (Table 1), we can calculate the fractal dimension for individual patches consisting of hexagonal grid cells.

$$D = 2\ln(P/3.722)/\ln(A)$$

## 4. Fractal dimension of a sub-landscape consisting of rectangular cells

For rectangular cells, the sides of a rectangle are of different lengths, making the calculation of the constant of proportionality $k$ more problematic. For instance, $k$ depends on the shape of the rectangular grid cell in question because this directly affects the area and perimeter calculations.

Let us consider the area $A$ of a single rectangular grid cell to be the basic unit (i.e., $A = 1$ as in Equation 4) and define $r$ as the ratio of the length of the longest side to the length of the shortest side:

$$r = l_{max}/l_{min} \qquad (7)$$

From Euclidean geometry we know that:

$$A = l_{max} \times l_{min} \qquad (8)$$

Solving for $l_{max}$ and $l_{min}$ we obtain:

$$l_{max} = \sqrt{r} \qquad (9)$$

$$l_{min} = 1/\sqrt{r} = \sqrt{r}/r \qquad (10)$$

From Equations 9 and 10, we can now calculate the perimeter $P$ in terms of $r$:

$$P = 2l_{max} + 2l_{min} = 2\sqrt{r} + 2(\sqrt{r}/r) = 2\sqrt{r}(1+1/r) \qquad (11)$$

giving

$$k = 2\sqrt{r}(1+1/r) \qquad (12)$$

So, for example, for a rectangular cell with a ratio $r = 2$ we would have (Table 2):

**Table 2. Proportionality constants (*k*) for rectangular pixels as a function of the ratio (*r*) of the length of the longest side to the length of the shortest side.**

| Ratio of sides | *k* |
| --- | --- |
| 1:1 | 4.000 |
| 2:1 | 4.243 |
| 3:1 | 4.619 |
| 4:1 | 5.000 |
| 5:1 | 5.367 |
| 10:1 | 6.957 |

$$k = 2\sqrt{2}\times(1+1/2) = 4.243$$

and for the simplest case of a square cell with $r = 1$ we would obtain:

$$k = 2\sqrt{1}\times(1+1/1) = 4$$

## 5. Conclusion

Generalizing the non-regression technique of Olsen et al. (1993) to handle raster databases consisting of any regular shaped grid cell allows natural resource managers and researchers to incorporate new GIS layers in the evaluation of landscape diversity using fractals.

Note that because the non-regression technique described here is intended for use on sampled areas of nominally scaled raster databases, the results of measuring the fractal dimension are obviously related both to the classification process and to the extent of the sub-landscapes sampled. Research related to the effects of classification process and sampling scheme on the distribution of diversity within the context of a larger landscape is currently underway.

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